Dynamics of vector solitons and vortices in two-dimensional photonic lattices

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We study discrete vector solitons and vortices in two-dimensional photonic lattices with Kerr nonlinearity and demonstrate novel types of stable, incoherently coupled dipoles and vortex–soliton complexes that can be excited by Gaussian beams. We also discuss what we believe to be novel scenarios of the charge-flipping instability of incoherently coupled discrete vortices. © 2006 Optical Society of America

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Nonlinear periodic photonic structures are attracting a lot of attention due to the unique ways they offer for controlling light,[1] opening up novel possibilities for light localization in the form of discrete optical solitons.[2] These nonlinear waves were introduced as spatially localized nonlinear modes of weakly coupled optical waveguides.[3] Similar effects can be studied in continuous periodic systems, where lattice solitons have been demonstrated experimentally for one-[4,5] and two-dimensional[6–9] (2D) optically induced photonic lattices (PLs), where novel types of self-trapped beams with a nontrivial phase have been found such as discrete optical vortices,[10,11] soliton dipoles,[12,13] and complexes.[14]

Although vector solitons were generated in continuum nonlinear systems,[2] their discrete analogs in 2D PLs have been observed only recently by Chen *et al.*,[15] who demonstrated both fundamental and dipolelike vector solitons by trapping two mutually incoherent beams in the lattice.

In this Letter we first analyze several types of incoherently coupled two-component discrete solitons and vortices in 2D PLs with a Kerr-type nonlinear optical response and demonstrate novel types of stable dipoles and vortex–soliton complexes. Second, we then reveal novel scenarios of the charge-flipping vortex instability in discrete lattices.[16,17] Thus we consider propagation of two mutually incoherent Gaussian beams with envelopes ψ_1 and ψ_2 launched in a nonlinear medium with a periodic modulation of the linear refractive index in the transverse direction. In dimensionless units the propagation is described by a system of two coupled nonlinear equations,[15]

$$i\frac{\partial \psi_j}{\partial z} + \Delta_\perp \psi_j + [V(x,y) + F(I)]\psi_j = 0, \qquad (1)$$

where $j=1,2$ and $F(I)=I=|\psi_1|^2+|\psi_2|^2$ is the nonlinear Kerr response, with the total intensity I normalized in units of $8a^2 n_0 n_2 |E|^2/\lambda^2$, a is the lattice spacing, n_0 and n_2 are the linear and nonlinear refractive indices, respectively, E is the field amplitude, and λ is the wavelength in vacuum. The propagation coordinate z is normalized by $4a^2/\pi\lambda^2$. The spatial variables x and y are scaled to a/π. $\Delta_\perp$ stands for the transverse Laplacian, $V(x,y)$ describes the transverse periodic modulation of the refractive index, and $V(x,y) = V_0 \cos(2\pi x/a)\cos(2\pi y/a)$.

First we analyze the generation of vector solitons[15] in model (1). Our interest in the Kerr model is twofold: first, it is possible to study it by approximate analytical methods; second, it can be considered as a first-order approximation of more complicated models as photorefractive systems that are widely used in experiments. Thus we consider input Gaussian beams of the form $\psi_1(r,0)=\psi_2(r,0)=A\exp(-r^2/w^2)$ and study the beam evolution numerically for different amplitudes with a split-step Fourier method. We find that for $0.8<A<1.2$ the two mutually incoherent beams evolve with very slight changes in their shapes and thus they form a stable on-site vector soliton on the lattice that holds over 100 times its Rayleigh length. Taking into account the normalization used, $A=1.0$ corresponds to a nonlinear phase shift of $n_2|E|^2=\lambda^2/8a^2n_0$ in physical units. We used amplitude potentials in the same range as the maximum nonlinear phase shift and beams of a size matching the lattice spacing to minimize radiation losses. We have found that slight variations around these values do not critically affect the stability of the structures. For large variations of the potential depth, we have observed that higher values of V_0 decrease the collapse threshold amplitude of the beam.

Then we place two mutually incoherent beams at the neighboring sites of the lattice, taking the initial conditions in the form $\psi_j(x,y,0)=A\exp\{[-[(x-x_j)^2+y^2]/w^2\}$, where (x_1,y) and (x_2,y) are the coordinates of the beams at different lattice minima, and we observe a periodic power exchange between the components (Fig. 1) for the unchanged total intensity, in full analogy with the effect of the induced soliton coherence observed for the planar soliton spiraling.[18]

To describe this effect analytically, we employ the variational approach[18] and present the dynamic solution in the form $\psi_1=A_{11}e_-+A_{12}e_+$ and $\psi_2=A_{21}e_-$

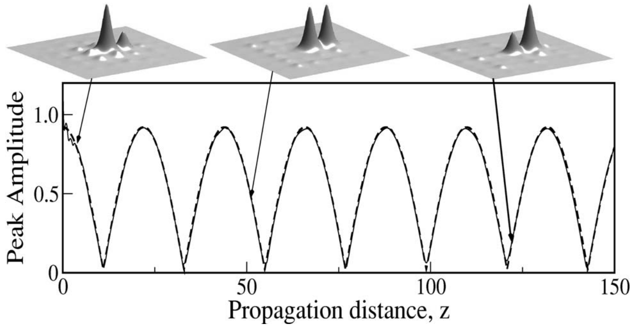

Fig. 1. Internal oscillations of the discrete vector solitons. Top: intensity of one component at three distances. Bottom: evolution of the normalized peak amplitude at one of the sites (solid curve, numerical; dashed curve, variational).

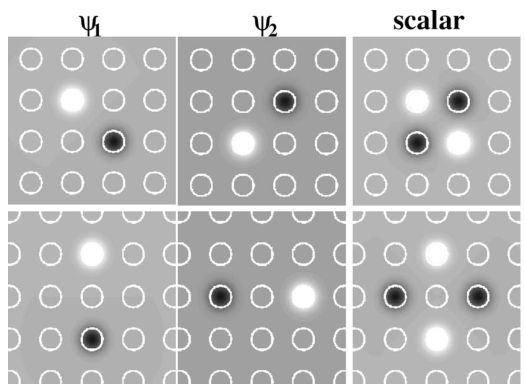

Fig. 2. Comparison between the cross-coupled vector dipoles and scalar quadrupoles of the same intensity for the off-site (top) and on-site (bottom) configurations.

$+A_{22}e_+$, where $e_\pm = \exp\{-[(x\pm x_0)^2+y^2]/2w^2\}$, and the complex amplitudes A_{ij} $(i,j=1,2)$ depend on z. Following the standard calculations and assuming a symmetric case, $A_{11}=A_{22}=A_+$ and $A_{12}=A_{21}=A_-$, we derive the conservation relation $|A_+|^2+|A_-|^2=N_1$, $A_+A_-^*+A_+^*A_-=N_2$, where N_1,N_2 are constants, and we obtain the following results: $|A_+|=A_0|\cos\Omega z|$ and $|A_-|=A_0|\sin\Omega z|$, where the spatial frequency Ω can be found in an explicit form.[19] In Fig. 1 we indicate the variational results with a dotted curve and note the excellent agreement with our numerical simulations.

For large distances between the input beams, the oscillation frequency is approximated by $\Omega \approx \frac{1}{2}N_1\exp(-x_0^2/w^2)$, and the period grows with the inverse of beam amplitudes, in agreement with the simulations. The oscillations are suppressed in the limit $A_+\approx A_-$, when the two-component input is known to create a stationary vector dipole soliton.[13] These dipoles can create a novel cross-coupled dipole soliton that is more stable than the apparently stable on-site and off-site quadrupoles.[13] Figure 2 compares the cross-coupled dipoles (the first two columns) with the scalar quadrupole soliton for both off-site and on-site configurations. Both solitons have the same intensity distribution, the cross-coupled dipoles being more robust, in contrast with similar modes in homogeneous systems.[20,21]

Next we study discrete vector vortices composed of two mutually incoherent off-site or on-site discrete vortices. First we show the results for the scalar discrete vortices of two types for different input amplitudes [Figs. 3(a) and 3(b)]. While the vortices are stable for low amplitudes, their phase structure is destroyed above a critical power for the off-site vortices. Charge flipping (detected by monitoring the phase during beam propagation) is observed for on-site vortices, as was discovered earlier.[16]

For mutually incoherent coupled fields, we find that two types of discrete vector vortices can exist, the two-component vortices with equal and opposite charges of the components, which we denote as (+1, +1) and (+1,−1), respectively. Their dynamics is shown in Figs. 3(c) and 3(d) structures for the off-site and Figs. 3(e) and 3(f) for the on-site structures. Quite remarkably, we observe, in sharp contrast with homogeneous media, that the (+1,−1) vector vortices with zero total charge can be stabilized by the lattice in the on-site geometry [Fig. 3(f)]. The vortex charge-flipping instability becomes very pronounced for the vector vortices [Figs. 3(d) and 3(e)], calling for the first experimental observations.

Finally, we couple several localized states in a lattice to create soliton–vortex complexes. Figure 4 shows the coherent vortex–soliton (top row) and vortex–vortex (bottom row) interaction when the neighboring solitons are out of phase. In all cases, no stationary complexes exist, and there is a strong charge-flipping dynamics. For mutually incoherent vortices with different radii, the interaction can be stabilized, as shown in Fig. 5, where oscillations of the power between neighboring sites are plotted in three different cases: (+1,0) ring with no charge (solid curve); (+1,+1) ring with same charge as the vortex (dotted curve); and (+1,−1) opposite charge ring (dotted–dashed curve). In all cases the vortex is reconstructed after the first oscillation period. The higher quality in the reconstruction is for the case of

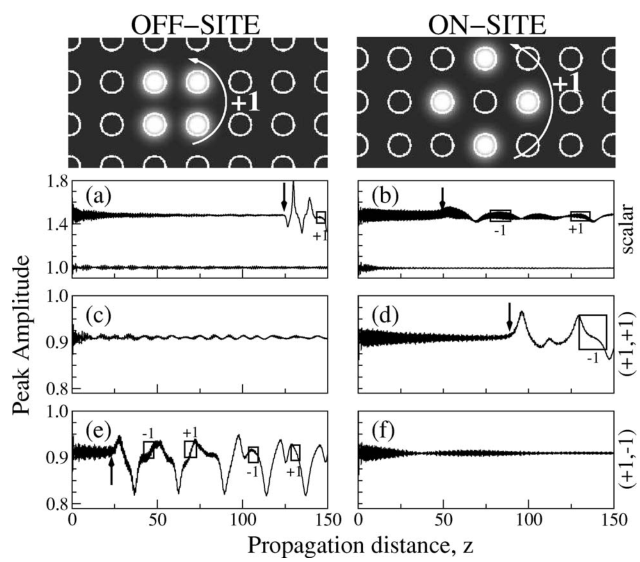

Fig. 3. Dynamics of off-site and on-site discrete scalar and vector vortices. (a)–(f) Change of the peak amplitude versus z for (a) and (b) scalar vortices, (c) and (d) equal-charge vortices, and (e) and (f) opposite-charge vector vortices. Arrows indicate the destruction of the vortex phase structure with the charge completely flipped marked by squares.

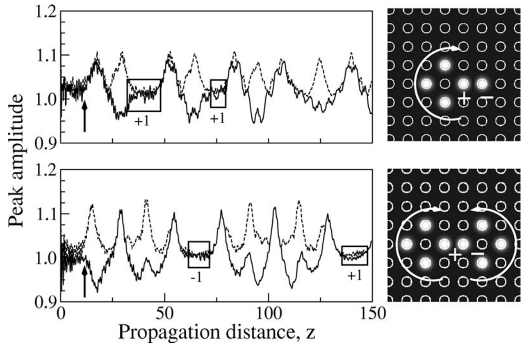

Fig. 4. Coherent vortex–soliton (top) and vortex–vortex (bottom) interaction when the neighboring solitons (indicated by + and −) are out of phase. Shown is the peak amplitude for the right soliton of the vortex (solid curve) and the soliton or the left soliton of the right vortex (dashed curve). Arrows indicate the destruction of the vortex phase.

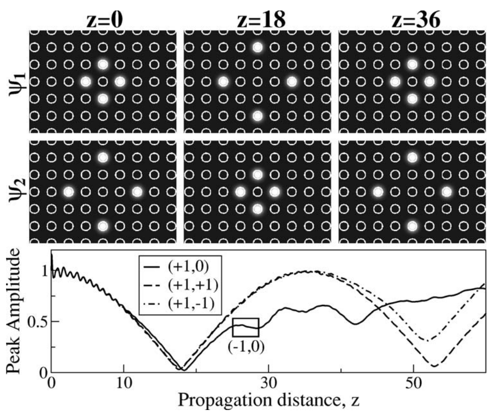

Fig. 5. Incoherent interaction of a vortex (top row) and soliton ring (bottom row) of three different charges.

same charges. In the $(+1,0)$ case the charge flips after revival (square window, bottom panel in Fig. 5).

In conclusion, we have introduced what we believe to be novel types of discrete vector solitons and vortices in photonic lattices. We have also revealed novel scenarios of the charge-flipping instability of discrete vortices.

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